

Universal scaling of the order-parameter distribution in strongly disordered superconductors

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We investigate theoretically and experimentally the statistical properties of the inhomogeneous order-parameter distribution (OPD) at the verge of the superconducting-insulator transition (SIT). We find within two prototype fermionic and bosonic models for disordered superconductors that one can identify a universal rescaling of the OPD. By performing scanning-tunneling microscopy experiments in three samples of NbN with increasing disorder we show that such a rescaling describes also with an excellent accuracy the experimental data. These results can provide a breakthrough in our understanding of the SIT.

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The interplay between disorder and superconductivity represents a typical example of emerging complex behavior in the presence of competing mechanisms. Indeed, while the former leads to localization of the electrons, leading to insulating-like transport, the latter favors the formation of a macroscopic coherent electronic state able to sustain a dissipationless current. While at moderate disorder level the pairing mechanism persists almost unchanged[1], as disorder increases the superconducting (SC) critical temperature T_c decreases and ultimately a full insulating state is reached. The most interesting case occurs when the superconductor-insulator transition (SIT) is somehow direct, i.e. without an intermediate bad-metallic state. Indeed, in this situation one can expect a persistence of SC correlations in the insulating state and conversely a precursor effect of the insulating order on the SC side[2, 3].

In the last few years considerable theoretical and experimental advances have been made to outline such a scenario on solid grounds. In particular, new insights have been offered by experiments of scanning tunneling microscopy[4–8], that have access to the local density of states (DOS) of homogeneously strongly disordered superconductors. The most striking features are the emergence of an intrinsic mesoscopic inhomogeneity in the local SC properties, and the occurrence of a large scale $\Delta_P \gg T_c$ for the DOS suppression, that persists well above T_c . These effects are understood qualitatively by using prototype models of disordered superconductors[9], that can be based either on a fermionic[10–16] or bosonic[17–19] description of the relevant degrees of freedom. In the former case it has been demonstrated the contribution of strong disorder to maintain the scale Δ_P finite at the SIT, driven essentially by a loss of SC phase coherence. In the latter case the focus has been put on a SIT driven mainly via a localization of

preformed pairs, due to quantum fluctuations associated to the random local energies. Within this scenario it has been also predicted[18, 19] a non-self-averaging behavior for the inhomogeneous order-parameter distribution, with a characteristic power-law decay which implies a diverging mean in the absence of the upper physical cutoff.

Despite some valuable attempts[6, 15] to establish a link between theoretical predictions and experiments, it has not been established yet which characteristic signatures of the SIT allow one for a convincing quantitative comparison between theory and experiments. The present work aims at filling this gap, by establishing at the same time a bridge between the two lines of theoretical investigations mentioned above. More specifically, in analogy with Refs. [6, 18, 19], we shall focus on the properties of the order-parameter distribution (OPD), in order to demonstrate the emergence of universal scaling properties at the verge of the SIT. We show that both within fermionic and bosonic models the OPD evolves at strong disorder not only with a diverging mean of the order parameter (OP), but also with a diverging width of the logarithm of the OP. This suggests an universal scaling of the OPD in the SC phase that covers not only the theoretical results obtained with different approaches, but more remarkably is in excellent agreement with experimental data in three different samples of disordered NbN films. We notice that such OPD differs from the one suggested in Refs. [18, 19]. Their result was obtained by a mapping into the directed-polymer model on the Cayley tree, where the number of neighbors grows exponentially with the distance, in contrast to ordinary finite-dimensional lattices. In this respect our result establish instead the relevance of the directed-polymer physics in finite dimension [21, 22] for the SIT.

The first prototype fermionic model for a disordered superconductor that we will analyze is the Hubbard

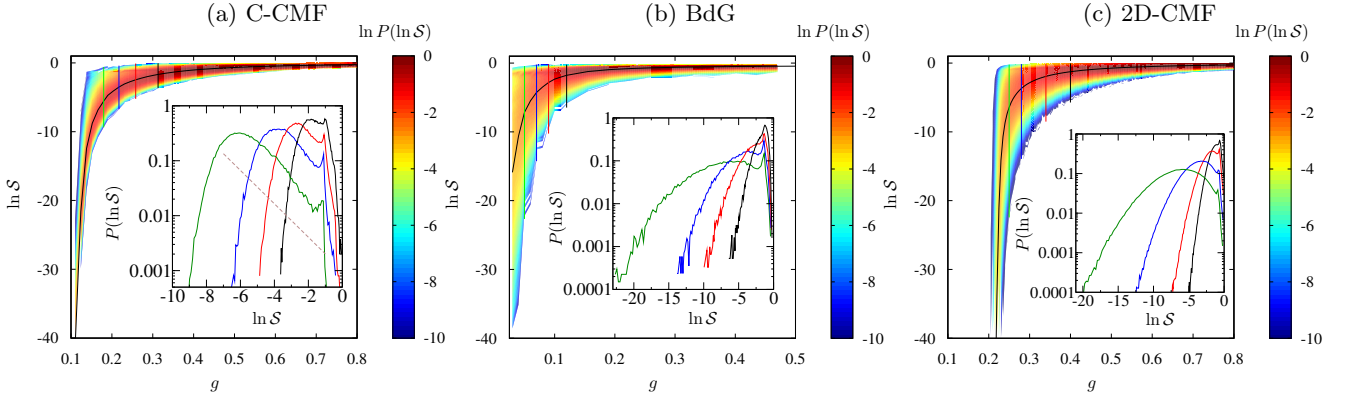


Figure 1: (Color online) Disorder-dependence of the OPD within (a) C-CMF, (b) BdG and (c) 2D-CMF. The probability for each S scales as the color code shown on the right of each panel. The maximum of the OPD is located approximately at the typical OP, S_{typ} , whose g dependence is shown with a continuous line in the main panels. The insets show explicitly the $\ln S$ dependence of the OPD for selected representative g values in the superconducting phase, marked by vertical bars in the main panels. Notice that in C-CMF a power-law behavior $P(\ln S) \sim S^{-0.7}$ sets in for large OP values, $S_{typ} \ll S \ll g/K$ as predicted in [19] (see the brown dashed line in the inset). Instead within BdG and 2D-CMF one observes the formation of strongly asymmetric distributions with large tails extending towards small S values. The parameters are the following (see text): (a) $L = 10$, (b) $|U| = 5$, $\langle n \rangle = 0.875$, (c) $L = 1000$.

model with random on-site energies:

$$H = -t \sum_{\langle ij \rangle, \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + \sum_{i, \sigma} (V_i - \mu) n_{i\sigma} - |U| \sum_i n_{i\uparrow} n_{i\downarrow}. \quad (1)$$

Here $c_{i\sigma}^\dagger$ ($c_{i\sigma}$) is the creation (destruction) operator for an electron with spin σ on a site \mathbf{r}_i of a square lattice with lattice spacing $a = 1$, $t = 1$ is the nearest-neighbor hopping, $|U|$ is the pairing interaction, $n_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma}$, and μ is the chemical potential. The on-site potentials V_i are independent quenched random variables box distributed between $-V_0$ and V_0 , with V_0 denoting the disorder amplitude. We will investigate the model (1) by means of Bogoliubov-de Gennes (BdG) mean field theory [11, 16, 23], by allowing for spatial fluctuations of the SC OP $\Delta_i \equiv |U| \langle c_{i\downarrow} c_{i\uparrow} \rangle$. Even though within such a mean-field approach one cannot describe the SIT, it captures already several features of strongly-disordered superconductors [11, 13, 24], as the emergence of spatial inhomogeneity of the OP and of a large DOS suppression due to the interplay between superconductivity and disorder. More recently it has been shown [16] also that at strong disorder the SC current follows a non-trivial percolative pattern, reminiscent of the glassy behavior suggested by the analysis of Ref. [18, 19]. Here we want to explore more deeply such analogy, by comparing the results obtained within the model (1) with the effective bosonic model investigated in Refs. [18, 19], i.e. the Ising spin model in a random transverse field:

$$H_I = - \sum_i \xi_i \sigma_i^z - g \sum_{\langle ij \rangle} \sigma_i^x \sigma_j^x. \quad (2)$$

The model (2) puts the main emphasis on the compe-

titition between local pairing and single-particle localization that is realized at strong disorder [10]. Thus, in this language a state with $\sigma_i^z = \pm 1$ corresponds to a site occupied or unoccupied by a Cooper pair, while the superconducting phase corresponds to the existence of a spontaneous magnetization in the x direction. The on-site energies ξ_i , which play the role of the transverse field in the spin language, are independent quenched random variables which are chosen here to be box distributed between -1 and 1 . The insulating phase corresponds thus to spins aligned along the z axis. The connection between the two approaches is suggested by a mapping between a disordered superconductor and a random (XY) spin model [10], where the low-energy degrees of freedom are pairs that can hop from one site to another. In the strong-coupling regime $U \gg t$ one can also show [25] that in the clean case the effective spin coupling is $g = t^2/V_0 U$. Notice that the model (2) lacks completely the XY symmetry that allows for the existence of the Goldstone mode, i.e. for the SC phase fluctuations. In this respect a comparison with the BdG approach, which also neglects phase fluctuations, is meaningful. In the following we will show that this approximation is enough to describe the anomalous effects of the OPD distribution at the verge of the SIT, since this physics is driven mainly by the competition between pair hopping and site localization. This does not exclude of course that at the SIT phase fluctuations will lead to additional remarkable effects, as discussed in Refs. [13–16] and suggested experimentally by measurements of the penetration depth [5].

The Hubbard model (1) has been investigated in a wide range of parameters and lattices of linear dimensions up to $L = 25$, with a large number of disorder configurations (up to 1920). As far as the Ising model (2) is concerned,

in addition to a standard mean-field approach (2D-MF) we used also a cavity-mean-field approximation[18–20], that allows one to include quantum effects which lead to a SIT for a finite value of the coupling g . To emphasize also the role of the underlying lattice we will show results for two different lattice structures: the Cayley tree with $K = 3$ (C-CMF), discussed in Refs. [18, 19], and the 2D lattice (2D-CMF) discussed in Ref. [21], where the cavity approximation is implemented via a non-linear transfer approach. For the random Ising model (2) defined on a Cayley tree of connectivity K , the C-CMF approximation [18, 19] describes the spin j by the local Hamiltonian $H_j^{\text{CMF}} = -\xi_j \sigma_j^z - \sigma_j^x \frac{g}{K} \sum_{k=1}^K \langle \sigma_k^x \rangle$ where $\langle \sigma_k^x \rangle$ is the magnetization at site k due to the rest of the spins *in absence of* j and $\tilde{g} \equiv Kg$ is the coupling parameter considered in this context and which we will continue to denote g in the following. By defining the cavity mean field $B_j = \frac{g}{K} \sum_{k=1}^K \langle \sigma_k^x \rangle$, one obtains at zero temperature the CMF recursion relation:

$$B_j = \frac{g}{K} \sum_{k=1}^K \frac{B_k}{\sqrt{B_k^2 + \xi_k^2}}. \quad (3)$$

The 2D-CMF approach propagates instead equation (3) with $K = 2$ along the diagonals of a 2D square lattice (see [21] for more details).

In Fig. 1 we show the evolution with the coupling parameter g of the OPD $P(\ln \mathcal{S})$ for C-CMF, BdG, and 2D-CMF. Here $\mathcal{S}_i = B_i/g$ is the normalized cavity field, which corresponds within BdG to a coarse-graining of the Δ_i over nearest neighbors, $\mathcal{S}_i = \frac{1}{4} \sum_{k=1}^4 \frac{2\Delta_k}{|U|}$. The probability at each $\ln \mathcal{S}$ value for the given disorder strength is represented in a color plot, where the maximum of the distribution is located approximately at the typical value of the OP, $\mathcal{S}_{\text{typ}} \equiv \exp(\ln \mathcal{S})$. In 2D-CMF and C-CMF the SIT transition occurs at $g_c \approx 0.22$ and $g_c \approx 0.11$, respectively, while in BdG the system remains superconducting right up to $g_c = 0$. As one can see in the insets of Fig. 1a, where $P(\ln \mathcal{S})$ is reported for some representative g values, for C-CMF we recover the expected power-law decay $P(\ln \mathcal{S}) \sim \mathcal{S}^{-m}$ with the universal (disorder-independent) exponent $m = 1 - eg_c \approx 0.7$ predicted in Refs. [18, 19]. However, such a power-law behavior is absent in the BdG and 2D-CMF results, where instead $P(\ln \mathcal{S})$ appears to be dominated by the low-field values, and to be strongly disorder-dependent. We notice that such a discrepancy between C-CMF and 2D results cannot be attributed to the method itself: indeed, as we discuss in the Supplementary Material, in 1D CMF and BdG give both $p(\mathcal{S}) \sim \mathcal{S}^\alpha$ with $\alpha \rightarrow -1^+$ when $g \rightarrow g_c$ which is in agreement with the exact critical behavior of the Ising model (2) [20, 22].

Such a distinction between C-CMF from one side and 2D results from the other can be made more quantitative by introducing as a scaling variable the logarithm of the OP, normalized to its variance $\sigma_{\mathcal{S}}^2 = \overline{\ln^2 \mathcal{S}} - \overline{\ln \mathcal{S}}^2$. In-

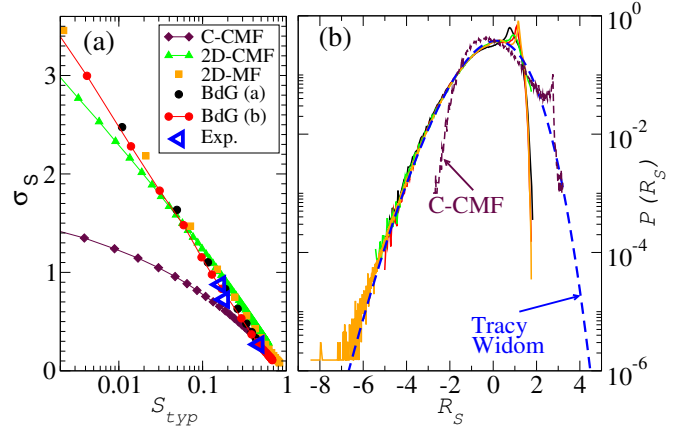


Figure 2: (Color online) (a) Evolution of the typical OP \mathcal{S}_{typ} and of the distribution width $\sigma_{\mathcal{S}}$ with increasing disorder (i.e. decreasing \mathcal{S}_{typ}) for 2D-CMF, 2D-MF, BdG and C-CMF. Notice that while within C-CMF $\sigma_{\mathcal{S}}$ saturates for increasing disorder the 2D results show all an increasing $\sigma_{\mathcal{S}}$. We also report the three points corresponding to the NbN samples analyzed in Fig. 3 below. (b) Rescaling of the OPD with respect to \mathcal{S}_{typ} and $\sigma_{\mathcal{S}}$. All the 2D results collapse in a single curve, well fitted by the Tracy-Widom distribution, while the C-CMF results follow a different behavior. The parameters are the following: BdG (a), $|U| = 9$, $\langle n \rangle = 0.3$, $L = 25$, $g = 0.1$; BdG (b), $|U| = 5$, $\langle n \rangle = 0.875$, $L = 25$, $g = 0.08$; 2D-CMF, $L = 1000$, $g = 0.4$; 2D-MF, $L = 120$, $g = 0.2$; C-CMF, $L = 15$, $K = 3$ and $g = 0.2$.

deed, as one can see in Fig. 2a, when disorder increases \mathcal{S}_{typ} and $\sigma_{\mathcal{S}}$ scale in the same way in the 2D case, while within C-CMF $\sigma_{\mathcal{S}}$ tends to saturate at strong disorder. This result hints to a remarkable property of the OPD, that becomes evident when the above data are rescaled with the variable $\mathcal{R}_{\mathcal{S}} = (\ln \mathcal{S} - \ln \mathcal{S}_{\text{typ}})/\sigma_{\mathcal{S}}$. As shown in Fig. 2b, provided that the coupling is small enough but still in the SC phase, all the data (except the ones for the C-CMF) collapse into a single curve, well approximated by the Tracy-Widom distribution[26], whose relevance in the insulating phase has been recently discussed in Refs. [21, 22]. Notice that such scaling holds for a wide range of parameters both within fermionic and bosonic models, showing the robustness of the present result.

To compare these results with experiments, we performed scanning tunneling spectroscopy (STS) measurements at 500 mK on three NbN films with different levels of disorder, having $T_c \sim 1.68$ K, 2.9 K and 6.1 K respectively. NbN seems to be the ideal candidate to investigate the SIT, since it has been shown earlier that its T_c decreases monotonically with increasing disorder, eventually giving rise to a non-SC state characterized by strong SC correlations[7]. The thickness of all the films was ~ 50 nm, which is much larger than the dirty-limit coherence length[28]. Details of sample deposition and characterization have been reported in Refs. [7, 29, 30].

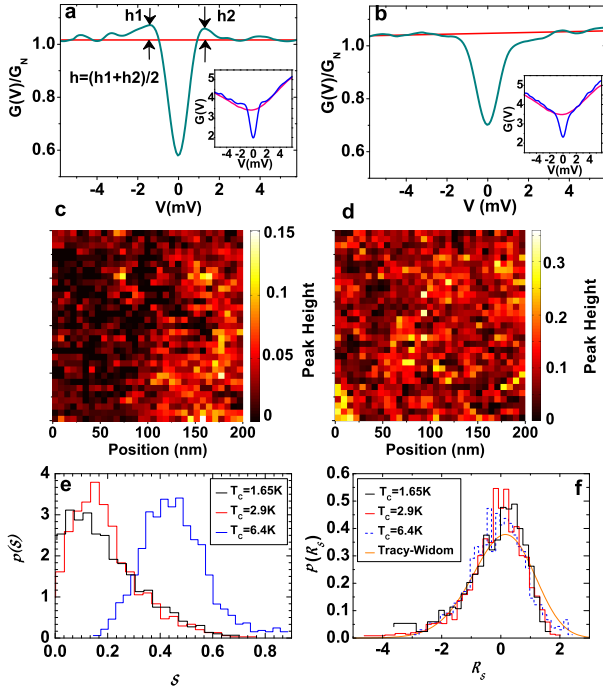


Figure 3: (Color online) (a)-(b) Representative background-corrected tunneling spectra at 500 mK for two different locations on a NbN film with $T_c \sim 1.65$ K; the horizontal red line shows the normal-state conductance. The insets show the corresponding tunneling spectra before background subtraction (blue line) along with the spatially-averaged spectrum at 8K used to subtract the background (red line). The average peak height $h \equiv (h_1 + h_2)/2$ measures the local order parameter, whose spatial variation measured at 500 mK is shown in panels (c) ($T_c \sim 1.65$ K) and (d) ($T_c \sim 2.9$ K) over a 200 nm X 200 nm area. Panel (e) shows the OPD in the three samples in linear scale. The same data are reported in panel (f) using the rescaling adopted in Fig. 2b. The theoretical curve corresponds to the Tracy-Widom distribution.

For STS measurements, the samples were grown in-situ in a sputtering chamber connected to a home-built scanning tunneling microscope (STM)[27]. For each film, tunneling conductance (dI/dV vs V) was measured at each point on a 32 X 32 grid over an area of 200 X 200 nm. All the spectra show a prominent dip in the conductance associated with the superconducting energy gap which rides over a broad V-shaped background arising from Altshuler-Aronov type electron-electron interactions which extends up to high bias (inset Fig. 3a, 3b). To isolate the feature associated with superconductivity from the background we follow the procedure adopted in Refs. [5, 7]: The spatially averaged tunneling spectrum recorded at 8 K where all superconducting correlations are destroyed is used to subtract the background from the spectra recorded at low temperatures. The background-corrected tunneling spectra do not show a significant variation in the magnitude of the superconducting energy gaps[31], but they show a large distribution in the height

of the coherence peaks: At some positions we observe a prominent coherence peak (Fig. 3a) whereas at other places the coherence peak is completely suppressed (Fig. 3b). We take the average of the heights of the coherence peaks at positive and negative bias from the normal state conductance values $h \equiv ((h_1 + h_2)/2)$ as the measure of the local OP Δ_i for our system[6, 15]. Figures 3c and 3d show the spatial variation of the OP in the form of an intensity plot for the samples with $T_c \sim 1.65$ K and $T_c \sim 2.9$ K, respectively. We observe smooth variation of Δ_i over length scales of few tens of nanometers, leading to the OPD for the three samples reported in Fig. 3e in linear scale. Here we defined $\mathcal{S} \equiv h/\text{Max}[h]$ to keep the same overall normalization used for the theoretical results. As disorder increases one observes a steady decrease of the OPD maximum along with a widening of the OPD, similar to the one reported in TiN samples in Ref. [6]. This can be further quantified by computing S_{typ}^{exp} and the variance of the log σ_S^{exp} , which are found to follow the same trend as the theoretical 2D results (see Fig. 2a). However the most striking result is that by introducing the scaling variable $\mathcal{R}_S = (\ln \mathcal{S} - \ln \mathcal{S}_{typ})/\sigma_S$ defined above the three experimental OPD distributions collapse into a single universal curve, despite their apparent difference when plotted in linear scale. In addition, the agreement with the universal Tracy-Widom distribution found in finite dimensions is also very good, even though it could be further improved at positive \mathcal{R} values to fully capture the striking universality of the experimental data.

In summary we have shown both theoretically and experimentally that the SC state at the verge of the SIT transition is characterized by an universal behavior of the OPD. The relevant scaling variable is the logarithm of the OP normalized to its variance. The latter diverges as the SIT is approached, unless the problem is studied on an infinite-dimensional lattice as the Caley tree, explaining the lack of such universality within the C-CMF[18, 19]. The universal OPD shares a pronounced similarity with the Tracy-Widom distribution, whose role in the disordered phase of the random Ising model has been recently discussed within the mapping into the directed-polymer model in finite dimensions[21, 22]. Within such a mapping additional predictions have been made, as e.g. the divergence of the dynamical critical exponent as the SIT is approached. This could be tested experimentally by the critical scaling of the superconducting fluctuations at T_c , as done recently in other systems[32]. While the critical properties of real systems at the SIT should ultimately belong to the XY universality class, at intermediate disorder further experimental and theoretical investigation of these predictions will further clarify the relevance of the directed-polymer physics on the SIT.

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